World Polarization*

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Abstract

Job polarization is a global phenomenon. I show this by extending the analysis of polarization from a group of developed countries to a sample of 119 economies. At all levels of development, employment shares in routine occupations have decreased since the 1980s. This suggests that routine occupations are becoming increasingly obsolete throughout the world, rather than being outsourced to developing countries. In order to study the technological trends behind this change, I propose a development accounting framework with technical change at the *task* level. This model allows me to quantify and extrapolate task-specific productivity levels. Recent technological change is biased against routine occupations and in favor of manual occupations. These trends imply that in the following decades, world polarization will continue: employment in routine occupations will decrease, and the reallocation will happen mostly from routine to manual occupations, rather than to abstract ones.

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1 Introduction

Job polarization is happening in the developed world. This is a fact that economists in labor and macroeconomics have documented.[1] In this paper, I present new stylized facts about the distribution of occupations in the world. With this, I expand current analyses of job polarization beyond a group of developed economies. I analyze a sample of 119 countries covering all levels of economic development to argue that job polarization is a global phenomenon.

Job polarization is concerned with the "hollowing out" of the employment distribution: occupations in the middle of the wage distribution decreasing their employment shares, while the shares for the occupations with higher and lower wages increase. These changes have been linked to the tasks that these occupations perform, resulting in three large groups: abstract, routine, and manual occupations. Abstract occupations mainly perform tasks requiring problem-solving, creativity, and persuasion typical of professional, managerial, and technical occupations. These occupations earn the highest wages, on average. Routine occupations perform tasks that follow well understood procedures, which makes them more susceptible to automation. Office clerks, and plant and machine operators are typical of this group of occupations, whose wages are in the middle of the wage distribution. The last group, manual occupations, perform tasks that require situational adaptability, visual recognition, and in-person interactions. Typical examples are occupations performing personal services, domestic and office cleaning, and construction and installation services.[2]

This polarization in labor markets has important implications for earnings, income distribution, and human capital. Job polarization is accompanied by wage polarization: wages in routine occupations have decreased compared to the other occupations (Firpo, Fortin & Lemieux, 2011). This polarizes the distribution of income, since routine occupations are in the middle of the wage distribution. Human capital has a strong occupation specific component (Kambourov & Manovskii, 2009). Therefore, changes in the occupational mix of the economy could lead to losses in accumulated human capital. These are all topics studied in the context of polar-

[1] See, for instance, Autor & Dorn (2013); Beaudry, Green & Sand (2016); Cortés (2016); Goos, Manning & Salomons (2014).

[2] For this paper, I borrow the classification from Cortés et al. (2014).

ization. The scope of this paper, however, is on the employment distributions, and their evolution over time.

The two main stylized facts that I uncover are as follows. First, at any point in time, there is a strong link between a country's development level (measured by its real income per worker) and its occupational employment shares. This is what I call the *occupational development profile*. Second, and more importantly, this profile has shifted over time, resulting in world polarization: a lower routine development profile, coupled with higher manual and abstract development profiles. These facts are important because they portray how modern economic growth has affected the world distribution of occupations. This confirms that polarization is a global phenomenon.

Put differently, world polarization implies lower employment shares in routine occupations at *all* levels of development, increasing those in manual and abstract occupations. Take the cases of Spain and Peru: In 1985, Spain had a real income level comparable to Peru in 2014. Back then, Spain had a routine employment share of 38 percent, while Peru, 30 years later, had an employment share 11 percentage points lower. The modern growth experience, then, is biased against routine occupations.

This finding contrasts to the results in the structural transformation literature. The link between income and employment shares is a stable one.[3] Industries and occupations are closely related, so it would be tempting to argue that the link between development levels and occupational employment shares should also be stable. The data in this paper reveals otherwise.

Having a global perspective on polarization is useful in discerning among its causes. The polarization analyses so far have identified two main explanations: international trade and technical change (see, for example Goos & Manning (2007) & Autor & Dorn (2013)). International trade, and outsourcing routine employment from developed to developing countries would decrease the routine employment share in developed countries, while increasing it in developing countries. Polarization is happening at all levels of development, and also within industries. This means that an explanation based on technical change fits the patterns in the data better.

To analyze these development patterns, I follow the grouping principle for occupations and develop a polarization accounting framework at the *task* level. This serves

[3] See Herrendorf, Rogerson & Valentinyi (2014) for a discussion.

two purposes. First, it allows to quantify the differences in task-specific productivities. This is important to determine the technological forces behind the occupational distributions. Second, it allows to study the process of technical change at a global level.

The main finding of this exercise is that technical change has been biased against routine occupations and biased in favor of manual occupations. Around the world, productivity growth has been highest in routine tasks, which makes workers to switch to different occupations. Productivity growth in abstract occupations has been the lowest, which goes in line with Baumol's cost disease (Baumol, 1967).

These technological trends imply that as countries continue to develop, we can expect lower employment shares in routine occupations, and higher in manual occupations, worldwide. Through a vector autoregression (VAR) analysis, I forecast the path of productivity growth, and conclude that world polarization will continue. In the following years, then, the development profile for routine occupations will keep decreasing, and the profile for manual occupations will keep increasing.

This paper is organized as follows. The first part discusses how the world distribution of occupations changed between 1980 and 2014. The second one analyzes these changes through a polarization accounting framework, and forecasts its changes until 2050.

2 World Polarization Trough the Lens of Occupational Development Profiles

To expand the sample of analyses of job polarization beyond the developed countries previously studied, I use two data sources with comparable occupational information: the International Labor Organization's ILOSTAT database, and census microdata from the IPUMS International Project. The result is an unbalanced panel of 119 countries, with an average period length of 19.4 years.[4]

The developed economies analyzed in Autor, Levy & Murnane (2003) and Goos, Manning & Salomons (2014) are relatively similar, which allowed for a simple comparison of employment shares without specifically taking into account their level of

[4]The details of the data handling process are left to appendix A. All the results I report are qualitatively similar for unbalanced panel that I discuss here in the main text and a balanced panel.

Table 1: Occupational Development Profiles: 1985 & 2014

	1985			2014		
	Abstract	Routine	Manual	Abstract	Routine	Manual
Constant	33.984***	48.285***	17.731***	42.660***	29.434***	27.906***
	(3.103)	(3.897)	(6.089)	(1.194)	(0.831)	(1.360)
Income	7.679***	4.369	-12.047**	10.762***	-0.963	-9.799***
	(2.284)	(2.868)	(4.481)	(1.234)	(0.859)	(1.406)
Income^2	0.417	-0.460	0.043	0.706***	-0.999***	0.294
	(0.370)	(0.465)	(0.727)	(0.260)	(0.181)	(0.296)
N	29	29	29	119	119	119
R^2	0.773	0.802	0.831	0.716	0.565	0.800

OLS estimates of equation (1) for 1985 & 2014. The dependent variable is the occupational employment share, and income levels are included as the base 2 logarithm of PPP-adjusted per-worker GDP, relative to U.S. levels in 2000. Asterisks indicate statistical significance at 10 percent (*), 5 percent (**), and 1 percent (***).
Source: author's calculations using ILOSTAT, IPUMS & PWT.

development. The countries I analyze are much more heterogeneous in their stages of development and occupational employment shares. In my broad sample it is thus important to study the link between the level of development and the occupational distribution of employment. To capture this link, I introduce the concept of the *occupational development profile* and use it to analyze polarization at the global level. The following subsection explains this concept in more detail.

Occupational Development Profiles

As countries develop, economic activity is reallocated across productive sectors (industries). This is one of the characteristics of modern economic growth, according to Kuznets (1973). More recent studies, like Duarte & Restuccia (2010) and Herrendorf, Rogerson & Valentinyi (2014), confirm that this reallocation still holds for industries across various countries.

Kuznets' observations, however, also refer to changes in the occupational status of labor. This is the counterpart of this stylized fact for the distribution of employment across occupations. With this in mind, I refer to *occupational development profiles* as the link between development levels and occupational employment shares.

More precisely, I define $d_{j,t}$, the occupational development profile in occupation j

and time t, as a function that relates a level of income y to its expected employment share in occupation j:

$$d_{j,t}(y) = \beta_{0,j,t} + \beta_{1,j,t}y + \beta_{2,j,t}y^2 \tag{1}$$

I estimate the parameters through ordinary least squares regressions, and table 1 presents the estimates for 1985 and 2014. Income levels correspond to the real, PPP adjusted GDP per worker relative to the U.S. level in 2000.[5] This measure is intended to be comparable over countries and time. Since development profiles reflect cross-sectional data, these are the snapshots, at a point in time, linking the development process to its occupational employment shares. For the regression analyses, this is expressed in logarithmic terms, base 2.

Figure 1 plots the estimated development profiles and employment shares for two years: 1985 and 2014. Each subpanel presents the information for an occupational group. The horizontal axes contain income levels, and the vertical axes occupational employment shares. Each circle represents a country, and its size is proportional to its level of employment. The information in black represents observations in 1985, and information in gold represents observations in 2014. It bears reminding that income is expressed relative to the U.S. level in 2000: during 1985, this income was 70 percent for the U.S., and 111 during 2014. This is to capture how occupational employment shares change during the different stages of development.

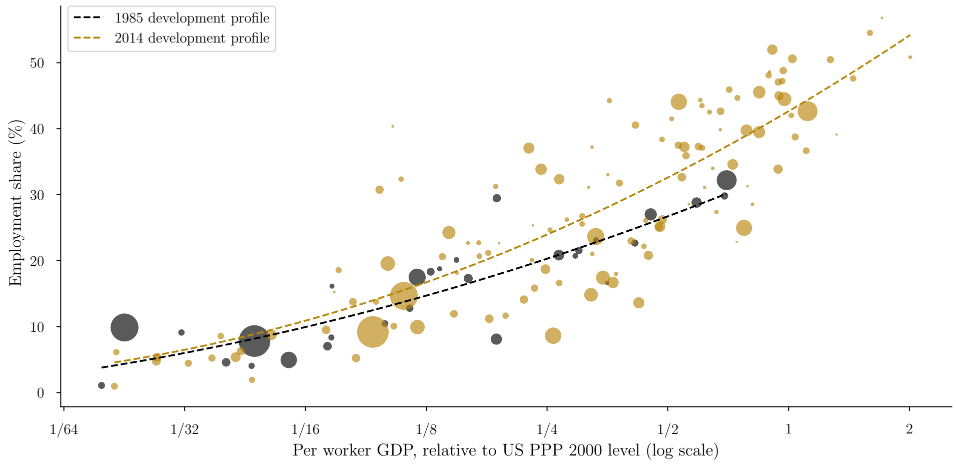
This figure shows that there is a strong link between a country's income level, and its occupational employment shares. Typically, a country with a low income level also has a low share of its employment in abstract occupations. Countries with higher incomes, also have higher employment shares in abstract occupations. This is the case during 1985 and 2014. Routine occupations show the same pattern; countries with low levels of income have low employment shares, and tend to increase them as they develop. Finally, manual occupations show the opposite pattern. Low levels of income are associated with very high shares in manual occupations.

These development profiles capture the link between the distribution of employment across occupations and income levels at a point in time. For a given year, the movements *along* these curves tell us how the occupational employment shares

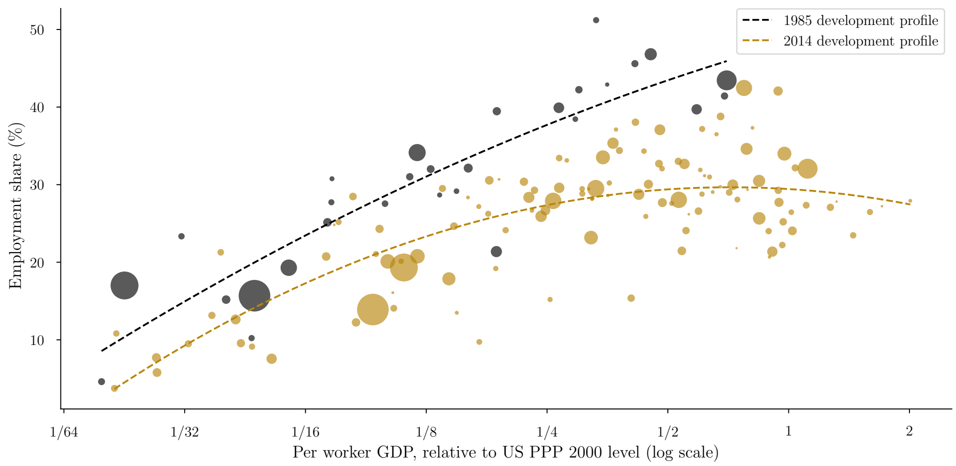
[5] This reference point is the same throughout the analysis. This means that the U.S. series will only have a level of 1 in 2000.

Figure 1: Occupational Development Profiles: 1985 & 2014

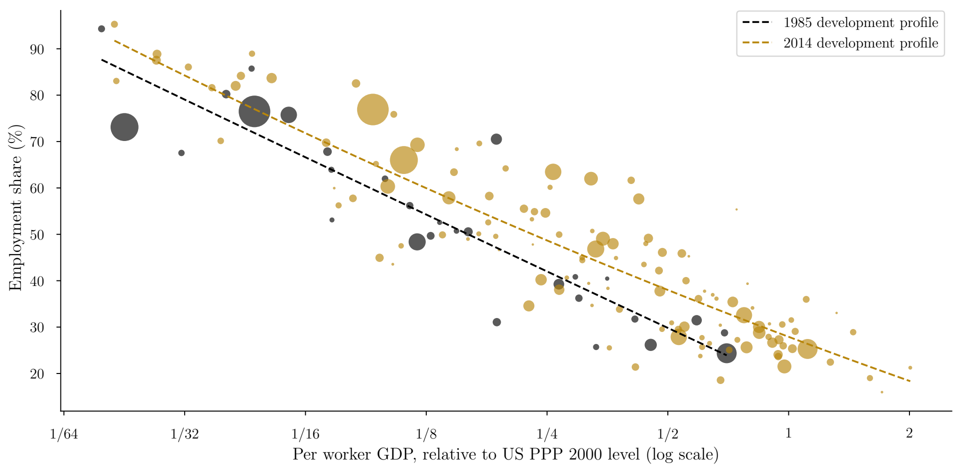
(a) Abstract occupations



(b) Routine Occupations



(c) Manual Occupations



Source: author's calculations using ILOSTAT, IPUMS & PWT.

are expected to change as countries develop. Take, for example, a country like Morocco. In 1985 its income level is 1/16 of the U.S. level in 2000, and its employment share in manual occupations was 68 percent. According to the development profile in 1985, if it were to develop and reach an income level of 1/8, we would expect it to decrease its employment share in manual occupations to 50. That's the expected employment share in manual occupations for a country with that level of income, like Chile. Therefore comparing countries with different levels of income at a point in time involves assessing movements *along* the occupational development profiles.

Despite this simple characterization, these profiles capture most of the variability in occupational employment shares. In 1985, for instance, the R-squared of these three regressions is 0.8 on average, as shown in table 1. In 2014, this average decreases to 0.7, mostly driven by the increased dispersion in routine occupational shares.

This analysis, so far, has focused on the employment changes for a given development profile. The next section analyzes the changes of these profiles change over time.

World Polarization

Job polarization, in developed economies, translates to decreases in employment shares for routine occupations. In the sample I use, between 1990 and 2014 the routine employment share for this group of economies (weighted by employment levels) decreased by 0.45 percentage points, annually. Abstract occupations increased its employment share by 0.41, while manual occupations increased it by 0.04.[6] This is consistent with the evidence in Autor, Levy & Murnane (2003), Autor & Dorn (2013), and Goos, Manning & Salomons (2014). World polarization, on the other hand, translates to decreases in the development profile for routine occupations, and increases in the development profiles of abstract and manual occupations.

This section focuses on the movements of the occupational development profiles, rather than individual changes in employment shares. Typically, countries with lower income levels have higher manual employment shares, and countries with higher income levels have higher abstract employment shares. The movements along the development profiles suggest different changes in their employment shares as they

[6]The list of developed economies is: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Great Britain, and United States.

develop. The countries I analyze are more heterogeneous, and are in different stages of development. Therefore, analyzing the changes in the entire profile provides a way to analyze employment trends, globally.

A second look at figure 1 shows how the world has polarized. For that, we need to compare the development profiles of 1985 (depicted by the black lines) to the profiles of 2014 (depicted by the golden lines). Analogously to the concept of job polarization, world polarization implies that the development profile of routine occupations decreased, while the development profiles of abstract and manual occupations increased.

Between 1985 and 2014, the largest change happened in the routine development profile. On average, every decade saw this development profile decrease by 3 percentage points. In 1985, a country with an income of 1/4 of the U.S. 2000 level, like Spain, had in expectation a routine employment share of 38 percent. In 2014, a country with the same income level, like Peru, had in expectation a routine employment share of 27.

This is a novel finding, which contrasts with the analyses in the structural transformation literature. Cross country evidence of employment changes at the *industry* level suggests that currently, a country with an income level of the U.S. in 1980 has a similar industrial composition to what the U.S. had in 1980.[7] Industries and occupations are closely related, but the connection between development and occupational employment levels changed over time.

Moreover, examining the relationship between polarization and structural transformation more closely reveals that polarization is happening mostly due to changes *within* industries, rather than *between* industries. Intuitively, the routine employment share in a country can decrease because it is moving towards industries that do not require routine workers (structural transformation, or changes between industries), or because all industries are requiring less routine workers (polarization within industries). A shift-share decomposition quantifies the contribution of these two movements. During these years, two thirds of the employment changes happened within industries. This means that polarization is happening in addition to structural transformation, not due to it. Further details of this shift-share analysis are presented in appendix B.

[7] See Herrendorf, Rogerson & Valentinyi (2014) for a discussion.

Notice that a lower routine development profile does not mean that all countries will have lower routine employment shares. It means that for a given level of development, routine employment shares will be lower, but as countries develop, they may have higher routine employment shares. The key distinction are the movements *along* the development profile, and the movements *of* the development profile. Take, for example, Costa Rica's development process. Between 1985 and 2014, its routine employment share barely increased: it went from 29.16 to 29.46 percent.[8] Its income level, however, did change: in 1985 it was 15 percent of the U.S. level in 2000, and by 2014 it had increased to 31. The development profile of 1985 predicted its routine employment share to increase by 7 percentage points. This corresponds to the movements along the development profile. In 2014, however, the development profiles had changed, and for Costa Rica's income level, it predicted a much lower routine employment share. This corresponds to the movements of the development profile. In practice, these two resulted in a slightly higher routine employment share.

Lastly, notice that the change in the routine development profile is different for countries with low and high income levels. It is not a parallel shift of the profile, and the counterparts of these movements are also different. For countries with lower income levels, this decrease is mostly made up for by a higher manual development profile, and for countries with higher income levels, in abstract. In figure 1, the change in the abstract development profile looks more like an increase in its slope, while the change in the manual development profile looks more like an increase in its intercept. This asymmetry implies that, as countries with low income levels develop, manual occupation shares will decrease, but more slowly than the development profile suggests.

To summarize the empirical findings, in this section I introduce the concept of the occupational development profiles. Its goal is to link the distribution of occupational employment shares to income levels. These development profiles have shifted over time, resulting in world polarization: a lower development profile for routine occupations, and higher profiles for manual and abstract occupations. This is, modern growth has been biased against routine occupations. The following section quantifies task-specific productivity levels that are compatible with these facts, and what their changes imply for the following years.

[8]During this period, most of the labor reallocation happened between abstract and manual occupations, exchanging about 6 percentage points.

3 Further Polarization Is on Its Way

The way an economy allocates its resources is informative of its productive technology. In this section, I apply this idea to the occupational distributions through a development accounting exercise. This allows to quantify the task-specific productivities behind occupational distributions, and document the biases in technological progress. This section is structured as follows. The first part introduces the polarization accounting framework, and the way to map it to the data. The second explains its results; how technical change has been biased taking as a benchmark the 1985 development profiles. The third part explains the VAR framework I use to forecast technological trends, and the fourth section analyzes these forecasts until 2050.

3.1 Polarization Accounting

The goal of this section is to provide a way to quantify two aspects of modern economic growth: the technological factors behind occupational employment shares, and their change over time. This is, I perform a polarization accounting exercise, similar to the development accounting discussed in Hsieh & Klenow (2010). Development accounting "uses cross-country data on output and inputs, at one point in time, to assess the contribution of differences in factor quantities and the efficiency with which these factors are used" (Caselli, 2005, p. 681). Polarization accounting uses data on occupational employment shares to quantify these efficiency levels.

For that purpose, consider an economy following the task-based approach to production, as in Acemoglu & Autor (2011). To produce output, firms need to combine tasks according to a production function. In that sense, tasks are the basic building blocks in production. To map this framework to the data discussed in the previous section, I assume there are three tasks: abstract (a), routine (r), and manual (m). For notation purposes, task-specific variables are denoted by the j subindex, so that $j \in \{a, r, m\}$.

The production of task j depends on two components: an amount labor, $l_{c,j,t}$, and a task-specific productivity level $A_{c,j,t}$. Their product results in efficiency units of labor. The subindex c represents the country, and t the time. Technology is labor-augmenting, and these task-specific productivity levels effectively capture all of the factors increasing these efficiency units. This is a reduced-form way of grouping factors like equipment and machines that are specific to the production of tasks, and capital stocks and total factor productivity that are not.[9]

Final output requires these three tasks. Production happens according to the following constant elasticity of substitution production function:

$$y_{c,t} = \left[\sum_{j \in \{a,r,m\}} \omega_j^{\frac{1}{\varepsilon}} \left(A_{c,j,t} l_{c,j,t} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

where $\varepsilon > 0$ is the elasticity of substitution among tasks, and $\omega_j > 0$ is the production intensity of task j. These task intensities add up to one.[10] In this framework, task-specific productivity levels and the distribution of labor are country-specific; the elasticity of substitution and the task intensities are not.

Labor can be allocated to the production of any of these tasks. For simplicity (and data limitations), labor is homogeneous and has no task specificity to it. Since the main interest is on the technological changes leading to polarization, I assume that labor is perfectly mobile across tasks.[11] I normalize the total labor force to 1, so that these labor inputs represent occupational employment shares. This means that $y_{c,t}$ effectively corresponds to output *per worker*. This is also the measure I match in the data. Then,

$$1 = \sum_{j \in \{a,r,m\}} l_{c,j,t} \tag{3}$$

I model the allocation of labor through competitive markets. Firms are price and wage takers, and there is free entry into the production of the final good. Their production technology is represented the production function (29), so that their

[9] An alternative, but equivalent approach would look at the costs of producing tasks, rather than its efficiency levels like Goos, Manning & Salomons (2014).

[10] These intensities are raised to the power $1/\varepsilon$ so that the limit case where $\varepsilon \to 0$ converges to a Leontief utility function: $\lim_{\varepsilon \to 0} = \min_{j \in \{a,r,m\}} \{\omega_j A_{c,j,t} l_{c,j,t}\}$.

[11] This is a common assumption in the structural transformation literature. See, for example, Baumol (1967), Kongsamut, Rebelo & Xie (2001), Ngai & Pissarides (2007), and Duarte & Restuccia (2010). Their focus, as in this paper, is on the technological aspects of employment movements, so this assumption is mostly made out of convenience. Other papers, like Caselli & Coleman (2001) and Bárány & Siegel (2018) consider labor heterogeneity. In a separate project, we analyze the costs of reallocation.

optimization problem is:

$$\max_{\{l_{c,j,t}\}_{j\in\{a,r,m\}}} \left[\sum_{j\in\{a,r,m\}} \omega_j^{\frac{1}{\varepsilon}} \left(A_{c,j,t} l_{c,j,t}\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} - \sum_{j\in\{a,r,m\}} w_{c,j,t} l_{c,j,t} \tag{4}$$

In this setting, the final good is the numeraire, and firms hiring labor face wages $w_{c,j,t}$. Free entry in this context implies wage equalization, since labor is homogeneous and mobility is costless. In addition, profits are zero due to constant returns to scale in production. The optimal allocation of labor depends on the production intensities, productivity levels, and elasticity of substitution as follows:

$$\frac{l_{c,j,t}}{l_{c,k,t}} = \frac{\omega_j}{\omega_k} \left(\frac{A_{c,k,t}}{A_{c,j,t}} \right)^{1-\varepsilon} \tag{5}$$

With this optimality condition and the normalization of the labor force, we can aggregate the production function. Final output can be expressed exclusively as function of the productivities in each of these tasks:

$$y_{c,t} = \left[\sum_{j\in\{a,r,m\}} \frac{\omega_j}{A_{c,j,t}^{1-\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \tag{6}$$

These last two expressions are key to matching the model with the data. For country c, we observe its income level $y_{c,t}$, and its distribution of employment by occupations, $\{l_{c,j,t}\}_{j\in\{a,r,m\}}$. The objective of this framework is to infer the task-specific productivities, $A_{c,j,t}$. Combining equations (5) and (6) allows to do so by setting a system of 3 equations in 3 unknowns:

$$A_{c,j,t} = y_{c,t} \left[\sum_{k\in\{a,r,m\}} \omega_j \frac{l_{c,k,t}}{l_{c,j,t}} \right]^{\frac{1}{1-\varepsilon}} \tag{7}$$

I set the elasticity of substitution ε to 0.35, as in Vindas (2017), which analyzes occupational U.S. data. In addition, I normalize income levels and task-specific productivities to U.S. levels in 2000, which provides a way to estimate the task-intensity parameters ω_j. Therefore, both income and task-specific productivities are expressed in relative terms to U.S. levels in 2000, maintaining consistency with the units of measurement of the empirical section.[12]

The last part of this article forecasts task-specific productivity growth as a way to forecast the development profiles. For that, we need an expression for the occupational employment shares as a function of the task-specific productivities. This results from combining equations (3) and (5):

$$l_{c,j,t} = \frac{\omega_j / A_{c,j,t}^{1-\varepsilon}}{\sum_{k \in \{a,r,m\}} \omega_k / A_{c,k,t}^{1-\varepsilon}} \tag{8}$$

3.2 Accounting Results

Countries develop and change their occupational employment shares according to their task-specific productivities. This section documents the productivity changes inferred from the polarization accounting exercise. The first part documents the historical changes by showing their growth rates. The second compares them to growth patterns implied by the development profiles of 1985.

Historical Growth

Table 2 shows the averages and the standard deviations of the productivity growth rates between 1985 and 2014. Each country-year pair is counted independently, so that countries with longer histories have a larger weight. It also presents these summary statistics, but weighted by their employment levels.

Between 1980 and 2014, productivity growth rates were the lowest in abstract occupations, independently of whether these are weighted by employment or not. This is a clear reflection of Baumol's cost disease, but applied to occupations (Baumol, Blackman & Wolff, 1985). Routine and manual occupations have very similar productivity growth rates. Productivity in routine tasks is slightly higher at the country level, but weighting by employment reverts this.

These changes in task-specific productivities relate to occupational employment shares through equation (5):

$$\frac{l_{c,j,t}}{l_{c,k,t}} = \frac{\omega_j}{\omega_k} \left(\frac{A_{c,k,t}}{A_{c,j,t}} \right)^{1-\varepsilon}$$

[12]This normalization plays no important role in the following analyses since it scales productivities by the same factor.

Table 2: World Task-Specific Productivity Growth Rates (%): 1980-2014

	N	Unweighted		Employment-weighted	
		Mean	Std. Dev.	Mean	Std. Dev.
Abstract	2186	1.027	7.732	1.885	5.287
Routine	2186	4.029	6.741	3.917	4.765
Manual	2186	3.840	6.901	3.933	4.573

Task-specific productivity growth rates estimated from equation (7), not average log-differences. Unweighted statistics consider each country-change pair equally; employment-weighted consider the country's employment levels
Source: author's calculations using IPUMS, ILOSTAT & PWT data.

Over time,employment shares will depend on the relative task-specific productivities, and their elasticity of substitution. In the production function, the magnitude of ε determines how employment shares respond to changes in relative productivities. When $\varepsilon > 1$, tasks in production are good substitutes: if $A_{c,j,t}$ increases, its employment share will do so as well. When $\varepsilon = 1$, the production function converges to a Cobb-Douglas, and employment shares do not depend on the productivities. When $\varepsilon < 1$, as I assume here, tasks are complements, and the production function resembles more a Leontief technology. When $A_{c,j,t}$ increases, this will actually decrease the employment share in occupation j.

These summary statistics are a good starting point to analyze how productivities changed over time. The nonlinearities in the production function mean that these effects will be different, depending on the productivity levels of the other tasks. The following section, then, analyzes productivity growth patterns taking into account the development profiles.

Measuring Productivity Biases

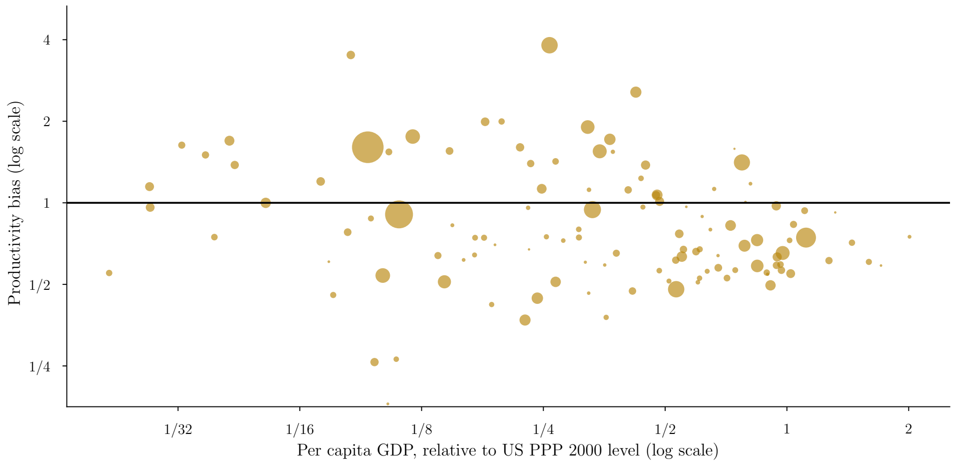
The development profiles summarize the link between levels of income and occupational employment shares in a given year. From equation (1):

$$d_{j,t}(y) = \beta_{0,j,t} + \beta_{1,j,t}y + \beta_{2,j,t}y^2$$

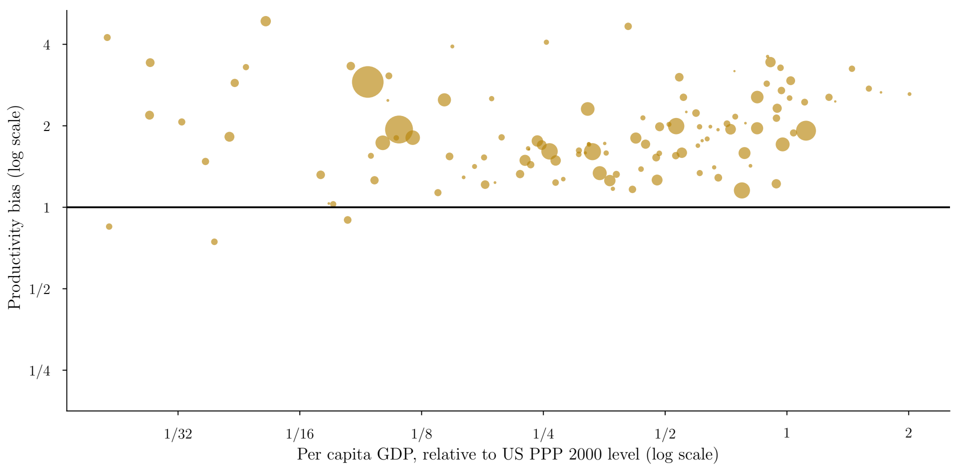
Therefore, these profiles summarize the expected path of occupational employment shares as countries develop. These also have a counterpart for the path of task-specific productivities. From equation (7), denote these expected productivity paths

Figure 2: Productivity Biases in 2014 with respect to 1985

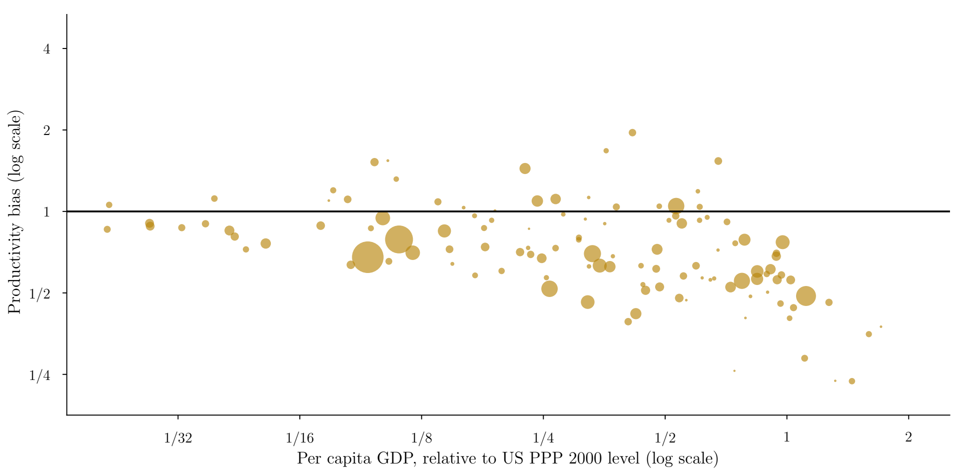
(a) Abstract occupations



(b) Routine Occupations



(c) Manual Occupations



Source: author's calculations using ILOSTAT, IPUMS & PWT.

by $\mathcal{A}_j$:

$$\mathcal{A}_j(y;t) = y\left[\sum_{k\in\{a,r,m\}} \omega_j \frac{d_{k,t}(y)}{d_{j,t}(y)}\right]^{\frac{1}{1-\varepsilon}}$$

Mechanically speaking, for all levels of income y, the development profiles $d_{j,t}(y)$ predict a distribution of employment shares, which altogether imply a level of task-specific productivity. The last section documented how these development profiles have shifted over time. Then, the changes in these paths of productivities provide a natural benchmark to evaluate whether technological growth has been biased.

Define the productivity bias in task j with respect to period t as the log-difference in a measured task-specific productivity level, A_j, with respect to the productivity level that the development profile of period t predicts for its income level, $\mathcal{A}_j(y;t)$. Denote this productivity bias by $b_j(y, A_j; t)$. Then,

$$b_j(y, A_j; t) = \log(A_j) - \log\left(\mathcal{A}_j(y;t)\right) \tag{9}$$

In this definition, productivity is biased against occupations j if $b_j(y, A_j; t) > 0$. This is, if task-specific productivity is higher than the development profile in period t predicted for the income level y. The bias is *against* occupation j because a higher task-specific productivity level implies a *lower* employment share. Conversely, productivity is biased in favor of occupation j if $b_j(y, A_j; t) < 0$ by the same reasoning.

Figure 2 shows the productivity biases in 2014 with respect to 1985. The horizontal axes show income levels, and the vertical axes the biases measured through (9). As before, each circle represents a country, and its size is proportional to its employment level.

Between 1985 and 2014, technical change has consistently been biased against routine occupations, and in favor of manual occupations. The bias in abstract occupations, shown in the first panel, follows no consistent pattern: for some countries it has been positive, and for some it has been negative. There does not seem to be any systematic relationship with respect to income levels, either. For routine occupations, the result is markedly different: productivity growth has been biased against these occupations because it has consistently been positive. Only in three countries the bias has been negative. Manual occupations show the opposite pattern: their productivity growth patterns result in negative biases, producing higher manual employment shares than the development profile of 1985 suggests. Furthermore, this bias shows a negative relationship with income, so that countries with higher levels of income have shown larger biases in favor of manual occupations. Then, technical change has consistently been biased against routine occupations, and in favor of manual occupations.

3.3 VAR Analysis

The previous sections focused on a historical analysis of task-specific productivities, by quantifying its growth patterns and biases. The following sections provide a forward-looking exercise by extrapolating these productivity trends, and analyzing the implications for employment distributions in the future.

A forecast of the world distribution of occupations could be based on the latest development profile, and predict employment changes *along* these profiles. This wouldn't be a satisfactory approach: the productivity biases between 1985 and 2014 suggest that movements *of* the development profiles are important in describing how the world distribution of occupations has changed. This means that an exercise in forecasting should focus on the evolution of the productivities, and their continued departure from the established development profiles. In this section, I analyze task-specific productivity growth through a VAR model. This is a flexible enough framework allowing for different growth rates across countries and task-specific productivities, as well as interaction terms.

The type of analysis that I use here has commonly been applied in studies of cross-country convergence of per capita GDP levels, like Barro & Sala-i-Martin (1992) and Caselli, Esquivel & Lefort (1996). This line of thought considers a negative relationship between initial income levels and their subsequent growth path. My analysis differs in two dimensions. First, instead of considering one level of income, I study a three-dimensional vector of productivities. These productivities relate to income levels through equation (6). Second, the productivities I study are expressed as *gaps* from a technological frontier. This follows the idea of technological diffusion in Parente & Prescott (1994).

In this particular setting, I use the U.S. productivities $A_{US,j,t}$ as the technological frontiers. I forecast future task-specific productivity levels in two steps. In the first,

Table 3: U.S. Productivity Log-Differences: 1980-2014

	Occupational Productivity Log-Difference		
	Abstract	Routine	Manual
Constant	0.0011	0.0488***	0.0236***
	(0.005)	(0.005)	(0.007)

OLS estimates of equation (11). Asterisks indicate statistical significance at 10 percent (*), 5 percent (**), and 1 percent (***). Source: author's calculations using IPUMS, ILOSTAT & PWT data.

I extrapolate historical trends for the U.S. productivity levels. In the second, I use a VAR model to predict the path of the gaps for each country relative to the U.S. These gaps are defined as:

$$\tilde{A}_{c,j,t} = \frac{A_{c,j,t}}{A_{US,j,t}} \tag{10}$$

The growth process of the U.S. productivities follows a simple path of constant growth rates. The model I estimate is:

$$\Delta \log(A_{US,j,t}) = \alpha_{US,j} + \epsilon_{US,j,t} \tag{11}$$

Which is a trio of random walks with drifts. Table 3 shows the results of regressing the difference of the U.S. log-productivity levels with only a constant term.

The average difference in the log-productivity of routine tasks is the highest at 0.049 log points per year, followed by manual at 0.024. These two are statistically significant. Productivity changes in abstract occupations are much lower at 0.001 log points per year. This is not statistically different from zero; its standard error is quantitatively similar to the other two, but its level is too low. These are the differences I use to forecast the technological frontier up to 2050. Between 2014 and 2050, productivity in abstract tasks will grow by a factor of 1.03, while in routine and manual the growth factor is 3.38 and 1.80, respectively.

The second step involves a VAR forecast of the productivity *gaps* with respect to the U.S. levels. Because I study the three productivities jointly, I group them into a vector of log-gaps: $\tilde{\boldsymbol{a}}_{c,t} = [\log \tilde{A}_{a,c,t}, \log \tilde{A}_{r,c,t}, \log \tilde{A}_{m,c,t}]'$. The model I estimate is:

$$\Delta \tilde{\boldsymbol{a}}_{c,t} = \boldsymbol{\alpha}_c + \mathbf{B}\tilde{\boldsymbol{a}}_{c,t-1} + \mathbf{e}_{c,t} \tag{12}$$

The dependent variables are the differences in the productivity log-gaps, $\Delta \tilde{\boldsymbol{a}}_{c,t}$. Countries can vary in terms of their institutions and resources, which can result in different long-run productivity gaps. I capture these differences through country fixed effects, grouped in the vector $\boldsymbol{\alpha}_c = [\alpha_{a,c}, \alpha_{r,c}, \alpha_{m,c}]$. The 3×3 matrix $\mathbf{B}$ contains the coefficients associated with the lagged log-gaps, and $\mathbf{e}_{c,t} = [\varepsilon_{a,c,t}, \varepsilon_{r,c,t}, \varepsilon_{m,c,t}]$ is the vector of error terms that are centered around zero, and are identically and independently distributed.

Two properties of this system are of interest: the long-run expected productivity log-gaps, and the dynamic path towards it. The long-run expected productivity log-gaps are the equivalent to a steady-state gap. These follow from equation (12), and require the expected differences in log-productivities to be zero. Therefore:

$$\tilde{\boldsymbol{a}}_{c,t} = \tilde{\boldsymbol{a}}_{c,t-1} = \bar{\boldsymbol{a}}_c \tag{13}$$

$\bar{\boldsymbol{a}}_c$ is the vector of long-run expected productivity log-gaps. This is equal to:

$$\bar{\boldsymbol{a}}_c = -\mathbf{B}^{-1}\boldsymbol{\alpha}_c \tag{14}$$

Notice these gaps have a common component through $\mathbf{B}$, but also depend on the country-specific fixed effects.

We now turn to the dynamic path towards these long-run log-gaps. For that, it is useful to rewrite equation (12) as deviations from the long-run productivity log-gaps. In expectation:

$$(\tilde{\boldsymbol{a}}_{c,t} - \bar{\boldsymbol{a}}_c) = (\mathbf{I}{+}\mathbf{B})(\tilde{\boldsymbol{a}}_{c,t-1} - \bar{\boldsymbol{a}}_c) \tag{15}$$

Then, the deviations from the long-run productivity log-gaps are fully determined by $\mathbf{I}{+}\mathbf{B}$. In particular, this implies that the forecast k periods in advance is:

$$(\tilde{\boldsymbol{a}}_{c,t+k} - \bar{\boldsymbol{a}}_c) = (\mathbf{I}{+}\mathbf{B})^k(\tilde{\boldsymbol{a}}_{c,t} - \bar{\boldsymbol{a}}_c) \tag{16}$$

This is the expression that I use to forecast the productivity log-gaps. Whether this system is convergent, and its speed of convergence depends on the eigenvalues of **I+B**.

Table 4 presents three components of the estimated VAR system. First, the estimates of the **B** matrix, second the eigenvalues of the matrix **I+B**, which dictate the dynamics over time, and lastly, the rates of convergence to the long-run expected productivity log-gaps.

Most of the estimates in **B** are statistically significant. The columns contain the estimates for each of the dependent variables, the differences of the log-productivity gaps. The rows group the effects of the independent variables, the lagged productivity log-gaps. The diagonal terms in abstract and routine productivities are negative, as expected. Heuristically, if we ignore the non-diagonal terms, countries with higher deviations from their long-run productivity log-gaps in abstract and routine tasks close these deviations faster. This is not the case for manual productivity, since its difference is associated with a positive, but not statistically significant, estimate of its lagged log-gap. This means that, log-gaps in manual productivity are mostly driven by log-gaps in routine productivity since it's the only coefficient that is statistically significant.

The dynamics of a growth regression are simple to interpret if the dependent variable is unidimensional. The model can be rearranged as a difference equation, and the coefficient associated to the first lag determines the speed of convergence to its steady state.[13] A three-dimensional vector requires a slightly different approach. The system (15) has to be diagonalized to get an autonomous system of difference equations. This simplifies greatly the analysis of its dynamic properties, and the steps to do so are explained in appendix C. This analysis boils down to two sets of values: the eigenvalues of the system, and their moduli. These are presented in the bottom part of table 4.

The system, altogether, is convergent with small cycles. The cyclical behavior follows from the eigenvalues, and the convergence from the moduli of these eigenvalues. Complex eigenvalues, like the ones from this system, indicate cyclical behavior. Convergence to the long-run log-gaps is not monotone, but the cyclical components (i.e., the coefficients associated to the imaginary part) are fairly small. The moduli

[13] See Barro & Sala-i-Martin (2004) for several applications of this analysis.

Table 4: VAR Estimate Results

Productivity log-gap in	Log-difference of productivity gap in Abstract	Routine	Manual
Abstract	-0.087***	0.015	-0.001
	(0.013)	(0.010)	(0.010)
Routine	-0.034***	-0.100***	-0.061***
	(0.009)	(0.010)	(0.009)
Manual	0.051***	0.050***	0.013
	(0.009)	(0.007)	(0.008)
Eigenvalues associated to the dynamic system			
0.915	0.955+0.019i	0.955-0.019i	
Annual convergence rates (moduli)			
	0.085	0.045	

Upper section contains the VAR estimate of matrix **B** in (12). Middle section contains the eigenvalues of matrix **I+B**, and lower section contains their moduli. Asterisks indicate statistical significance at 10 percent (*), 5 percent (**), and 1 percent (***).
Source: author's calculations using IPUMS, ILOSTAT & PWT data.

of these eigenvalues are less than one, which mean that the system is convergent to the long-run productivity log-gaps in (14). Notice that these convergence rates correspond to the diagonalized system, which is a linear combination of the three productivity log-gaps. These converge at a rate between 4.5 and 8.5 percent per year. The non-diagonalized convergence rates, the ones at the log-gap level, are discussed in the following section. These are lower, and closer to the estimates of Sala-i-Martin (1994).

3.4 Forecasting Results

The previous section analyzed the growth patterns of task-specific productivities through a VAR model. In this one, I use the estimated model to extrapolate them. This exercise forecasts historical trends for 36 years, and the main goal is to describe the occupational development profiles in 2050. The forecasts begin in 2015, since current income data in the Penn World Tables is published up to 2014.

Even though the main goal is to describe employment shares, this exercise forecasts task-specific productivities. Employment shares, as described in equation (8)

Table 5: U.S. Task-Specific Log-Productivity & Occupational Employment Shares (1980, 2014 & 2050)

Year	Log-Productivity			Employment Shares		
	Abstract	Routine	Manual	Abstract	Routine	Manual
1980	15.305	14.509	14.662	29.42	45.94	24.64
2014	15.341	16.167	15.465	42.66	32.06	25.29
2050	15.379	17.926	16.316	56.90	19.71	23.39

Historical data for 1980 & 2014; 2050 forecasts from model (11).
Source: author's calculations using IPUMS, ILOSTAT & PWT data.

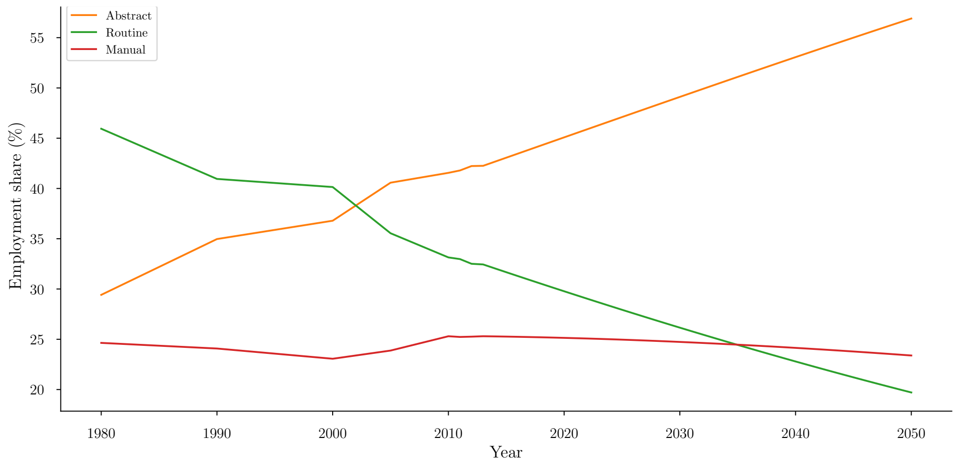
are non-linear functions of the countries' task-specific productivities. Because of that, I focus on predicting these productivities as a way to construct occupational employment shares. This, as explained previously, involves two steps. In the first one, I forecast task-specific productivity growth in the U.S., which is the technological frontier in this framework. The second one forecasts the gaps with respect to that frontier. Ultimately, I combine these two forecasts to get employment shares, and describe the occupational development profiles.

Technological Frontier Forecast

The results of the productivity extrapolation in the U.S. are in table 5. It contains the task-specific log-productivities and their implied employment shares in 1980, 2014, and 2050. Naturally, we can compare two periods. The first contains historical U.S. data from 1980 to 2014, and the second contains the forecasted data from 2014 to 2050.

By construction, log-productivities in the forecasted years change at the same rate as the historical data. The occupational employment shares, by contrast, change differently. The employment share in routine occupations continues to decrease, but its change slows down. During the historical period it decreased by 0.41 percentage points, annually, and in the forecasted it decreases by 0.34. The employment share in manual occupations goes from minor increases to minor decreases. During the historical period it increased by 0.02 percentage points annually, and decreases by 0.05 in the forecasted data. The increases in the employment share in abstract occupations slightly accelerate from 0.39 percentage points, to 0.40 in the forecasted data. These differences are due to the non-linearities that determine the employment shares, even

Figure 3: U.S. Occupational Employment Shares: 1980 - 2050



Source: author's calculations using ILOSTAT, IPUMS & PWT.

though the underlying productivities grow at constant rates. These shares are also plotted in figure 3. By the end of the forecasted period, routine occupations will have the lowest employment share. Manual occupations will decrease during these years, but routine occupations will decrease at a much faster pace.

Productivity Gaps Forecast

The second step forecasts the differences in log-productivity gaps with the VAR model. As discussed in the previous section, the dynamics imply a convergent path towards long-run productivity log-gaps, which are specific to each country in the sample. Table 6 shows the results of this forecast. To summarize the information at the world level, it reports averages across countries. First, it contains the productivity log-gaps with respect to the technological frontier in 2014 and 2050. These follow from the definition in equation (10): $\tilde{\boldsymbol{a}}_{c,t} = [\log \tilde{A}_{a,c,t}, \log \tilde{A}_{r,c,t}, \log \tilde{A}_{m,c,t}]'$. It also shows the log-deviation from the long run productivity gaps: $\tilde{\boldsymbol{a}}_{c,t} - \bar{\boldsymbol{a}}_c$. The last column shows the average change over this period, which is the same for the log-gaps and the log-deviations. Log-gaps show how far countries are from the technological frontier; log-deviations show how far countries are from their long-run productivity gaps.

The larger log-gaps are in the productivity of manual tasks, followed by routine and abstract tasks. These log-gaps close over time, and maintain the same order

Table 6: World Productivity Gaps: 2014 & 2050

Occupation	Log-gaps		Log-deviations		Average change
	2014	2050	2014	2050	
Abstract	-0.503	-0.013	-0.637	-0.142	0.014
Routine	-1.231	-0.519	-1.016	-0.299	0.020
Manual	-3.007	-1.961	-1.363	-0.309	0.029

Log-gaps as defined in equation (10), and log-deviations as in (15). Averages correspond to the simple mean across countries; average change is the same for log-gaps and log-deviations.
Source: author's calculations using IPUMS, ILOSTAT & PWT data.

by the end of the forecasted period. This is true as well for the log-deviations from the long-run gaps. Notice, however, that these log-deviations are not negligible by the end of the forecasted period. For abstract occupations, a log-deviation of -0.142 translates to a ratio of its productivity gap with respect to its long-run productivity gap of 0.9.[14] This is, by 2050, the average productivity gap in abstract tasks will be at 90 percent of its long-run value. For routine and manual tasks, these values will be 81.3 and 80.7 percent. This means that the deviations from the long-run productivity gaps will still remain an important determinant of the employment distributions. The last column shows the average change during the forecasted period, which is the same for the log-gaps and the log-deviations. Overall, convergence to the long-run log-gaps is fastest in manual occupations, followed by routine and abstract.

Employment Shares Forecast

The forecast of the productivity log-gaps and the productivity frontier provides enough information to forecast the employment distributions up to 2050. For that, I use equation (8):

$$l_{c,j,t} = \frac{\omega_j / A_{c,j,t}^{1-\varepsilon}}{\sum_{k \in \{a,r,m\}} \omega_k / A_{c,k,t}^{1-\varepsilon}}$$

Table 7 shows yearly average changes in occupational employment shares for two periods: the historical period between 1980 and 2014, and the forecasted period between 2014 and 2050. To summarize the changes in the world distribution of

[14]These are base 2 logarithms, so that elevating 2 to the power $\tilde{a}$ results in the ratio of the productivity levels.

Table 7: World Annual Occupational Employment Share Changes: 1980-2014 & 2014-2050

Occupation	Country average (unweighted)		World average (weighted)	
	1980-2014	2014-2050	1980-2014	2014-2050
Abstract	0.346	0.397	0.195	0.294
Routine	-0.193	-0.243	-0.082	-0.211
Manual	-0.154	-0.154	-0.113	-0.083

Changes expressed as percentage points per year. Country averages weight each country-year change equally; weighted averages use employment in 2014.
Source: author's calculations using IPUMS, ILOSTAT & PWT data.

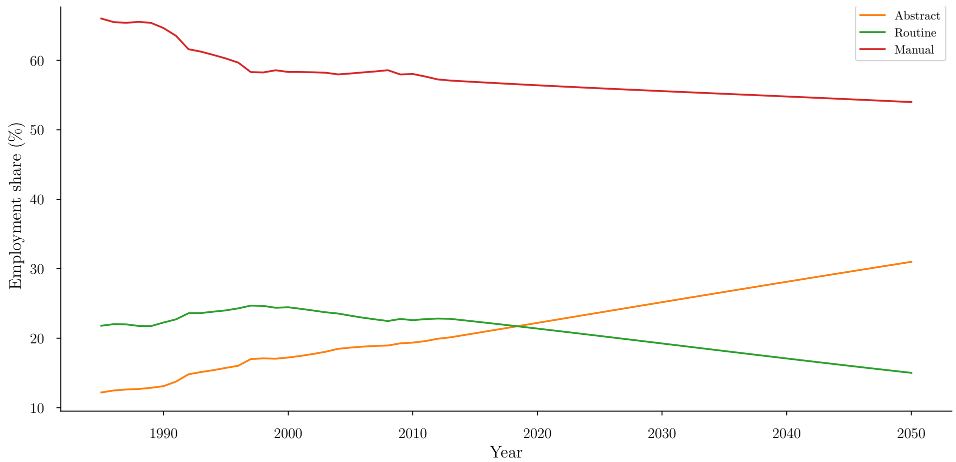
occupations, it shows two averages: one that weights equally each observation, and another one that weights them according to the countries' employment levels in 2014.

The occupational changes observed during the historical period will continue until 2050. Abstract occupations will continue to increase, while routine and manual will continue to decrease. Whether we look at the simple averages, or the employment-weighted ones, the results are very similar. Worldwide, the employment-weighted share in abstract occupations will increase, and will accelerate over time. The historical data shows that it increased by 0.195 percentage points every year, and the forecasted shares increase by 0.294 percentage points per year. The worldwide decrease in routine occupations will accelerate quite dramatically. From decreasing 0.082 percentage points annually, the forecast establishes that it will decrease 0.211 in the following years. Finally, the drop in manual occupations will slow down. Historically, it fell by 0.113 percentage points annually, but the forecasts suggest it will drop by 0.083 annually. These changes may seem small at face value, but represent enormous amounts of workers reallocating. Over the course of the forecasted period, 226 million workers will reallocate to abstract occupations, 162 millions will move out of routine occupations, and 64 millions will move out of manual occupations.

Graphically, these changes are plotted in figure 4. It shows the occupational employment shares, weighted by the countries' employment levels, between 1985 and 2050.[15] At the world level, abstract occupations increase and manual occupations decrease monotonically, but routine occupations show a "hump-shape". This is reminiscent of the analyses of structural transformation (employment shares by industry,

[15]This plot begins in 1985 since between 1980 and 1984, employment for the countries in the sample reached less than 65% of the level of the full sample.

Figure 4: World Occupational Employment Shares: 1985 - 2050



Source: author's calculations using ILOSTAT, IPUMS & PWT.

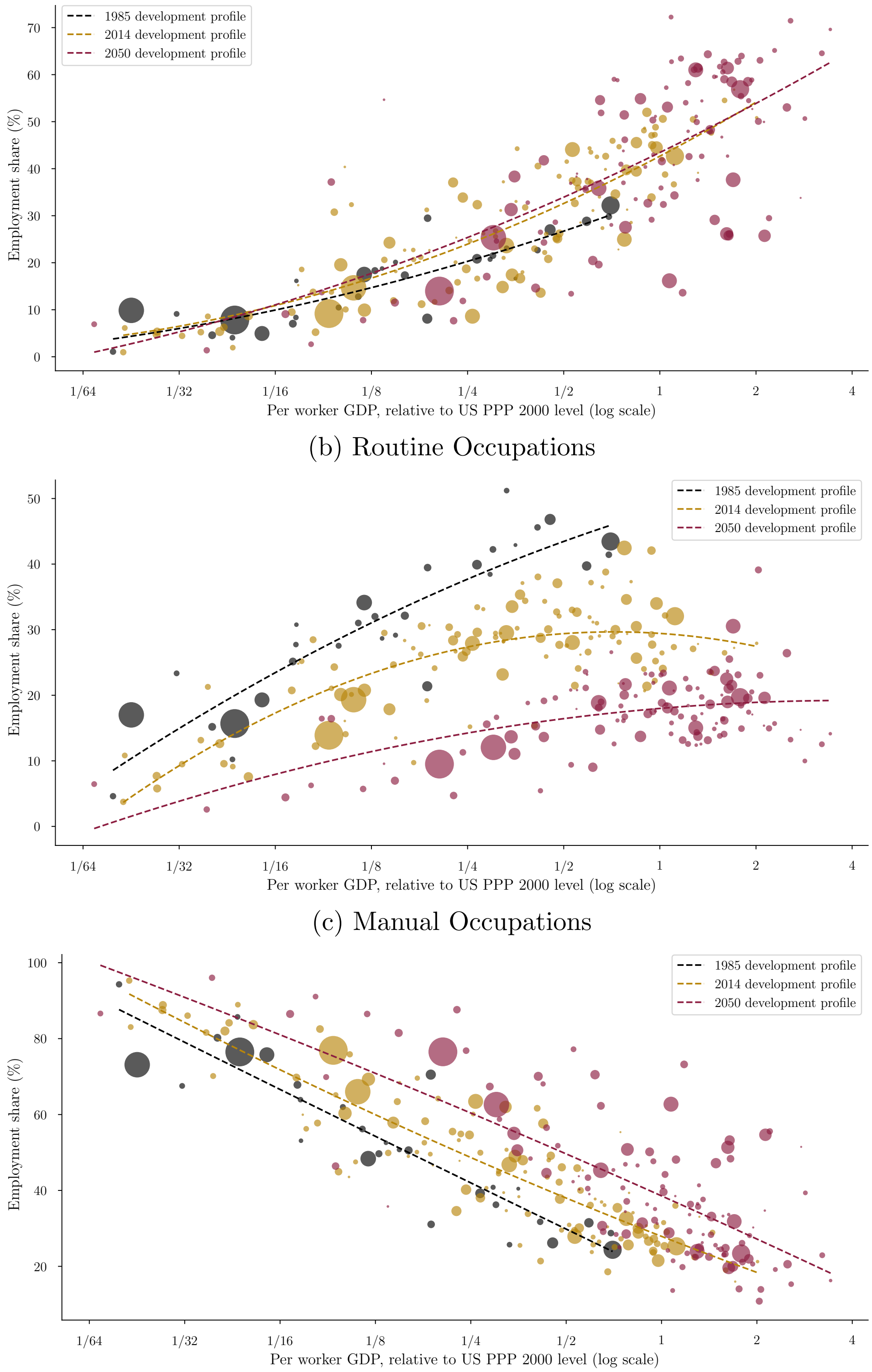
rather than occupation), like Ngai & Pissarides (2007), Buera & Kaboski (2009), and Duarte & Restuccia (2010). Differently to these, this plot aggregates the employment shares at the world level, instead of the country level data. Qualitatively, it shares the rise and fall of one of the categories, in this case routine occupations. The world employment share in abstract occupations will increase from 20 percent in 2014 to 31 in 2050. Routine occupations will decrease from 23 percent to 15, and manual occupations will decrease slightly from 57 to 54 percent.

The last paragraphs described the predicted changes at the world level, aggregating over countries. These countries, however, will still differ in their levels of income and employment distributions. Precisely due to these differences, this paper analyzed the occupational development profiles, rather than world totals. Figure 5, then, shows the predicted development profile of 2050, in addition to the development profiles of 1985 and 2014. As before, the observations in black represent data from 1985, and the observations in gold represent data from 2014. The forecasted data for 2050 is presented in maroon.

The main result is that world polarization will continue. Between 1985 and 2014, this meant a lower development profile for routine occupations, and higher development profiles for abstract and manual occupations. Between 2014 and 2050, this will be the case as well: the development profile in abstract occupations will be slightly higher, in routine occupations it will be much lower, and in manual

Figure 5: Occupational Development Profiles: 1985, 2014 & 2050

(a) Abstract occupations



Source: author's calculations using ILOSTAT, IPUMS & PWT.

occupations it will be higher. Most of the changes will happen among these last two occupations.

Perhaps surprisingly, the development profile of abstract occupations changes little, compared to the change between 1985 and 2014. The shape of the profile in 2050, in the first panel, shows only a slight increase. This is a reflection of the biases documented in figure 2. In abstract tasks, productivity growth showed no systematic bias. Because of that, the development profile changed little.

The development profile in routine occupations, in the second panel, keeps decreasing. It also shows a larger drop than between 1985 and 2014. This means that in the following years, productivity growth in routine tasks will be high enough to make many routine workers redundant. It's also worthy to note how the "hump-shape" flattens over time. During the forecasted period, countries will catch up to the technological frontier, that is making routine tasks very productive. This causes technological progress in the rest of the countries biased against routine occupations, resulting in a flatter profile.

For manual occupations, in the third panel, changes happen in the opposite direction. The development profile shifts upwards, which means that for a given level of income, a country will have on average a higher employment share in 2050 compared to both 1985 and 2014. The reasoning behind this change in the development profile, is the same as for routine occupations. The productivity growth bias, however, happens in the opposite direction.

These results are driven completely by the growth patterns of task-specific productivities. These, as explained before, are built in two steps: one forecasts the evolution of the technological frontier, and the other one forecasts the gaps with respect to that frontier. Which one is driving most of the employment shifts? Over time, we can decompose overall productivity changes into these two components:

$$\Delta \boldsymbol{a}_{c,t} = \Delta \boldsymbol{a}_{US,t} + \Delta \tilde{\boldsymbol{a}}_{c,t} \tag{17}$$

This tells us how much of the changes in productivity is due to the frontier growing, and how much by the gaps closing. Table 8 shows this decomposition. The first column shows the average annual change in the task-specific productivities, $\Delta \boldsymbol{a}_c$, and breaks it down into the contribution of the technological frontier, $\Delta \boldsymbol{a}_{U.S.}$, and

Table 8: World Productivity Growth Sources

	Average log-productivity change (annual)	Average contribution (%)	
		Frontier	Gaps
Abstract	0.013	17.2	82.8
Routine	0.067	74.2	25.8
Manual	0.050	55.2	44.8

Results of decomposition (17). Averages correspond to unweighted means across countries.
Source: author's calculations using IPUMS, ILOSTAT & PWT data.

the change in countries' gaps, $\Delta\tilde{\boldsymbol{a}}_c$.

The contribution of these two factors, overall, is sizable. This decomposition, however, throws very different results across tasks. For abstract tasks, most of the growth is due to the gaps closing, while in routine tasks, the technological frontier growth outpaces the contribution of the gaps closing. For manual tasks, the growth in the productivity still plays a predominant role. This means that the obsolescence of routine workers is mostly a product of the productivity changes happening at the technological frontier. Productivity growth is so high that its effect will eventually be transmitted to the rest of the countries, and push routine employment shares down, as in this forecast exercise.

4 Conclusions

In this paper, I present new stylized facts about the distribution of occupations in the world. I expand significantly the countries in the analysis to a sample of 119 countries covering all levels of economic development. The result of this analysis is that job polarization is a global phenomenon.

At any point in time, there is a strong link between a country's development level and its occupational employment shares. This is what I call the *occupational development profile*. Over time, this profile has shifted, resulting in world polarization. The development profile for routine occupations has decreased, coupled with higher manual and abstract development profiles. The modern growth experience, then, is biased against routine occupations.

To analyze these development patterns, I follow the grouping principle for occupations and develop a polarization accounting framework at the *task* level. Technical change has been biased against routine occupations and biased in favor of manual occupations.

These technological trends imply that as countries continue to develop, we can expect lower employment shares in routine occupations, and higher in manual occupations, worldwide. Through a vector autoregression (VAR) analysis, I forecast the path of productivity growth, and conclude that world polarization will continue. In the following years, then, the development profile for routine occupations will keep decreasing, and the profile for manual occupations will keep increasing.

In this paper, I present a first analysis of the task-specific productivities behind world polarization. Future research plans include the expansion of this model to endogenize technical progress through an *task-investment-specific* growth model. The details of such a framework are presented in appendix D, and its implementation is left as future work.

References

Acemoglu, Daron, & David Autor. 2011. "Skills, Tasks and Technologies: Implications for Employment and Earnings." , ed. O. Ashenfelter & D. Card Vol. 4 of *Handbook of Labor Economics*, Chapter 12, 1043–1171. Elsevier.

Autor, David, & David Dorn. 2013. "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market." *American Economic Review*, 103(5): 1553–97.

Autor, David, Frank Levy, & Richard Murnane. 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration." *The Quarterly Journal of Economics*, 118(4): 1279–1333.

Barro, Robert J., & Xavier Sala-i-Martin. 1992. "Convergence." *Journal of Political Economy*, 100(2): 223–251.

Barro, Robert, & Xavier Sala-i-Martin. 2004. *Economic Growth.* . Second ed., MIT Press.

Baumol, William. 1967. "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis." *The American Economic Review*, 57(3): 415–426.

Baumol, William, Sue Anne Batey Blackman, & Edward N. Wolff. 1985. "Unbalanced Growth Revisited: Asymptotic Stagnancy and New Evidence." *The American Economic Review*, 75(4): 806–817.

Beaudry, Paul, David Green, & Benjamin Sand. 2016. "The Great Reversal in the Demand for Skill and Cognitive Tasks." *Journal of Labor Economics*, 34(S1): S199–S247.

Buera, Francisco J., & Joseph P. Kaboski. 2009. "Can Traditional Theories of Structural Change Fit the Data?" *Journal of the European Economic Association*, 7(2-3): 469–477.

Bárány, Zsófia L., & Christian Siegel. 2018. "Job Polarization and Structural Change." *American Economic Journal: Macroeconomics*, 10(1): 57–89.

Caselli, Francesco. 2005. "Chapter 9 Accounting for Cross-Country Income Differences." In . Vol. 1 of *Handbook of Economic Growth*, , ed. Philippe Aghion & Steven N. Durlauf, 679 – 741. Elsevier.

Caselli, Francesco, Gerardo Esquivel, & Fernando Lefort. 1996. "Reopening the Convergence Debate: A New Look at Cross-Country Growth Empirics." *Journal of Economic Growth*, 1(3): 363–389.

Caselli, Francesco, & John Coleman. 2001. "The U.S. Structural Transformation and Regional Convergence: A Reinterpretation." *Journal of Political Economy*, 109(3): 584–616.

Cortés, Guido Matias, Nir Jaimovich, Christopher Nekarda, & Henry Siu. 2014. "The Micro and Macro of Disappearing Routine Jobs: A Flows Approach." National Bureau of Economic Research, Inc NBER Working Papers 20307.

Cortés, Guido Matías. 2016. "Where Have the Middle-Wage Workers Gone? A Study of Polarization Using Panel Data." *Journal of Labor Economics*, 34(1): 63 – 105.

Duarte, Margarida, & Diego Restuccia. 2010. "The Role of the Structural Transformation in Aggregate Productivity." *The Quarterly Journal of Economics*, 125(1): 129–173.

Firpo, Sergio, Nicole Fortin, & Thomas Lemieux. 2011. "Occupational Tasks and Changes in the Wage Structure." Institute for the Study of Labor (IZA) IZA

Discussion Papers 5542.

Goos, Maarten, & Alan Manning. 2007. “Lousy and Lovely Jobs: The Rising Polarization of Work in Britain.” *The Review of Economics and Statistics*, 89(1): 118–133.

Goos, Maarten, Alan Manning, & Anna Salomons. 2014. “Explaining Job Polarization: Routine-Biased Technological Change and Offshoring.” *American Economic Review*, 104(8): 2509–26.

Greenwood, Jeremy, Zvi Hercowitz, & Per Krusell. 1997. “Long-Run Implications of Investment-Specific Technological Change.” *American Economic Review*, 87(3): 342–362.

Herrendorf, Berthold, Richard Rogerson, & Ákos Valentinyi. 2014. “Growth and Structural Transformation.” In *Handbook of Economic Growth.* Vol. 2 of *Handbook of Economic Growth*, Chapter 6, 855–941. Elsevier.

Hsieh, Chang-Tai, & Peter J. Klenow. 2010. “Development Accounting.” *American Economic Journal: Macroeconomics*, 2(1): 207–23.

Kambourov, Gueorgui, & Iourii Manovskii. 2009. “Occupational Specificity Of Human Capital.” *International Economic Review*, 50(1): 63–115.

Kongsamut, Piyabha, Sergio Rebelo, & Danyang Xie. 2001. “Beyond Balanced Growth.” *The Review of Economic Studies*, 68(4): 869–882.

Kuznets, Simon. 1973. “Modern Economic Growth: Findings and Reflections.” *The American Economic Review*, 63(3): 247–258.

Minnesota Population Center. 2018. “Integrated Public Use Microdata Series, International: Version 7.1 [Dataset].” *Minneapolis, MN: IPUMS.*

Ngai, Rachel, & Christopher Pissarides. 2007. “Structural Change in a Multisector Model of Growth.” *American Economic Review*, 97(1): 429–443.

Parente, Stephen L., & Edward C. Prescott. 1994. “Barriers to Technology Adoption and Development.” *Journal of Political Economy*, 102(2): 298–321.

Sala-i-Martin, Xavier. 1994. “Cross-sectional regressions and the empirics of economic growth.” *European Economic Review*, 38(3): 739 – 747.

Vindas, Alberto. 2017. “Job Polarization, Structural Transformation and Labor Force Participation.” Arizona State University Working Paper.

Appendix A Data Sources

Table 9: Country data availability

Country	Years	Country	Years
Argentina	2004–2014	Armenia	2011–2014
Aruba	1994–2011	Australia	1991–2014
Austria	1981–2014	Azerbaijan	2003–2014
Bahamas	1991–2009	Bahrain	1991–2004
Barbados	1994–2014	Belarus	1999–2009
Belgium	1993–2014	Belize	1993–1999
Bermuda	2000–2007	Bhutan	2006–2014
Bolivia	1992–2014	Botswana	1991–2011
Brazil	1980–2014	Bulgaria	2000–2014
Burkina Faso	1985–1996	Cambodia	1998–2008
Canada	1981–2011	Cayman Islands	1991–2008
Chile	1982–2014	China	1982–1990
Hong Kong SAR	1994–2014	Costa Rica	1984–2014
Croatia	1996–2014	Cyprus	1999–2014
Czech Republic	1993–2014	Denmark	1992–2014
Dominica	1991–2001	Dominican Republic	1981–2014
Ecuador	1982–2014	Egypt	1986–2014
El Salvador	2008–2012	Estonia	1990–2014
Fiji	1986–2007	Finland	1997–2014
France	1982–2014	Georgia	1998–2007
Germany	1992–2014	Ghana	1984–2010
Greece	1981–2014	Guatemala	2012–2014
Guinea	1983–1996	Haiti	1982–2003
Hungary	1990–2014	Iceland	1991–2014
India	1983–2012	Indonesia	1980–1995
Iran	1996–2014	Ireland	1981–2014
Israel	1995–2014	Italy	1992–2014
Jamaica	1982–2008	Japan	2009–2014
Kazakhstan	2001–2013	Kyrgyzstan	2003–2014
Latvia	1996–2014	Lebanon	2004–2007
Lithuania	1997–2014	Luxembourg	1992–2014

Table 10: Country data availability (ctn'd)

Country	Years	Country	Years
Malawi	1987–2008	Malaysia	1980–2014
Mali	1987–2014	Malta	2000–2014
Mauritius	1995–2014	Mexico	1990–2014
Mongolia	2005–2014	Montenegro	2005–2014
Morocco	1982–2011	Mozambique	1997–2007
Namibia	2000–2014	Nepal	1999–2008
Netherlands	1992–2014	New Zealand	1992–2008
Nicaragua	1995–2014	Norway	1996–2014
Pakistan	2002–2014	Panama	1980–2014
Paraguay	1982–2014	Peru	1993–2014
Philippines	2001–2014	Poland	1995–2014
Portugal	1991–2014	Republic of Moldova	1999–2014
Romania	1995–2014	Russian Federation	1997–2014
Saint Lucia	1994–2006	Sao Tome and Principe	2003–2012
Senegal	1988–2002	Serbia	2004–2014
Seychelles	2011–2014	Singapore	1985–2014
Slovakia	1994–2014	Slovenia	1995–2014
South Africa	1996–2014	Spain	1981–2014
Sri Lanka	2002–2014	Suriname	2004–2014
Sweden	1997–2014	Switzerland	1980–2014
Macedonia	2002–2014	Taiwan	1994–2013
Thailand	2001–2013	Trinidad and Tobago	1980–2014
Turkey	1985–2014	Turks and Caicos Islands	2002–2007
Tanzania	1988–2012	Uganda	1991–2003
Ukraine	1999–2014	United Kingdom	1991–2014
United States	1980–2014	Uruguay	1996–2014
Venezuela	1981–2001	Vietnam	1999–2014
Yemen	1999–2014	Zambia	2000–2010
Zimbabwe	2011–2014		

Appendix B Shift-share details

This section documents how world polarization happens mostly within industries. We can express the share of each occupation as a weighted average at the country level. For period t:

$$l_{c,j,t} = \sum_{I} s_{c,t}(I) l_{c,j,t}(I) \tag{18}$$

where $l_{c,j,t}$ is the country's employment share of occupation j, $s_{c,t}(I)$ is the country's employment share of industry I, and $l_{c,j,t}(I)$ is the share of occupation j within industry I. The change between period 0 and t can be decomposed into its *between* and *within* industry components:

$$\Delta l_{c,j,t} = \underbrace{\sum_{I} \Delta s_{c,t}(I) \bar{l}_{c,j,t}(I)}_{\text{Between industries effect}} + \underbrace{\sum_{I} \Delta l_{c,j,t}(I) \bar{s}_{c,t}(I)}_{\text{Within industries effect}} \tag{19}$$

$\bar{l}_{c,j,t}(I)$ is the average between time 0 and t of the conditional occupation share, and and $\bar{s}_{c,t}(I)$ that of the industry share. The *between* effect refers to the impact of structural transformation, the changes in the productive structure of the economy. The *within* effect refers to the occupational mix inside each industry.

Both the changes within- and between-industries contribute to polarization. The within-industry changes dominate: on average, the within-industry contribution accounts for 63% of polarization.

Appendix C VAR Convergence System

The VAR model analyzes the productivity *gaps* with respect to the U.S. levels. Remember these gaps are defined as:

$$\tilde{A}_{c,j,t} = \frac{A_{c,j,t}}{A_{US,j,t}}$$

The vector of log deviations from the U.S. is $\tilde{\boldsymbol{a}}_{c,t} = [\log \tilde{A}_{a,c,t}, \log \tilde{A}_{r,c,t}, \log \tilde{A}_{m,c,t}]'$, and the econometric model I use is

$$\tilde{\boldsymbol{a}}_{c,t} - \tilde{\boldsymbol{a}}_{c,t-1} = \boldsymbol{\alpha}_c + \mathbf{B}\tilde{\boldsymbol{a}}_{c,t-1} + \mathbf{e}_{c,t} \tag{20}$$

$\boldsymbol{\alpha}_c = [\alpha_{a,c}, \alpha_{r,c}, \alpha_{m,c}]$ captures country fixed-effects, $\mathbf{B}$ is a 3×3 matrix containing the β coefficients associated with the lagged log-deviations, and $\mathbf{e}_{c,t} = [\varepsilon_{a,c,t}, \varepsilon_{r,c,t}, \varepsilon_{m,c,t}]$ is the vector of error terms.

This system implies a long-run expectations for the gaps in each of the countries

$$\tilde{\boldsymbol{a}}_{c,t} = \boldsymbol{\alpha}_c + (\mathbf{I}{+}\mathbf{B})\tilde{\boldsymbol{a}}_{c,t-1} \tag{21}$$

The dynamics are dictated by the eigenvalues associated to $\mathbf{I}{+}\mathbf{B}$. When all eigenvalues are unique, the system can be transformed into three autonomous difference equations. Matrix $\mathbf{I}{+}\mathbf{B}$ can be diagonalized as follows:

$$\mathbf{D} = \mathbf{V}^{-1}(\mathbf{I}{+}\mathbf{B})\mathbf{V} \tag{22}$$

where $\mathbf{D}$ is a diagonal matrix containing the eigenvalues, and $\mathbf{V}$ is an invertible matrix with the associated eigenvectors. If we define, $\mathbf{z}_{c,t} = \mathbf{V}^{-1}\tilde{\boldsymbol{a}}_{c,t}$, the system can be rewritten as:

$$\begin{aligned}\mathbf{z}_{c,t} &= \mathbf{V}^{-1}\tilde{\boldsymbol{a}}_{c,t} \\ &= \mathbf{V}^{-1}\boldsymbol{\alpha}_c + \mathbf{V}^{-1}(\mathbf{I}{+}\mathbf{B})\mathbf{V}\mathbf{V}^{-1}\tilde{\boldsymbol{a}}_{c,t-1} \\ &= \boldsymbol{\zeta}_c + \mathbf{D}\boldsymbol{z}_{c,t-1}\end{aligned} \tag{23}$$

This is a system of autonomous difference equations. If the j-th row has a real-valued eigenvalue (λ_j), then

$$\begin{aligned}z_{j,c,t} &= \zeta_{j,c} + \lambda_j z_{j,c,t-1} \\ \Rightarrow z_{j,c,t} &= \frac{\zeta_{j,c}}{1-\lambda_j} + \left(z_{j,0} - \frac{\zeta_{j,c}}{1-\lambda_j}\right)\lambda_j^t\end{aligned} \tag{24}$$

This equation is convergent when $|\lambda_j| < 1$, so that the last term goes to 0 as $t \to \infty$. $\zeta_{j,c}/(1-\lambda_j)$ is the steady-state level, and λ_j determines how fast deviations from it converge. Therefore, $1 - \lambda_j$ is the percentage of the gap with respect to the steady-state level that is closed each period, or the convergence rate.

If the j-th and $j+1$-th rows have complex-valued eigenvalues of the form $\lambda_j = \alpha + \theta i$, $\lambda_{j+1} = \alpha - \theta i$, then the homogeneous parts have form

$$z_{j,c,t} = r^t(A_1 \cos \omega t + A_2 \sin \omega t) \tag{25}$$

$$z_{j+1,c,t} = r^t(B_1 \cos \omega t + B_2 \sin \omega t) \tag{26}$$

where $r^2 = \alpha^2 + \theta^2$ is the modulus, and defines the convergence rate.

Appendix D A Model of Task Investment Specific Technical Change to Understand Polarization

The goal of this model is to understand two aspects of modern economic growth: the technological factors behind occupational employment shares, and their change over time. This fits naturally into an investment specific technical change framework, which needs to be extended to include the different tasks.

D.0.1 Economic Environment

Consider an economy populated by a representative household. Its preferences are given by:

$$U = \sum_{t=0}^{\infty} \beta^t \log(c_t) \tag{27}$$

where c_t represents consumption during period t, and $\beta > 0$ is the discount factor.

This economy follows the task based approach to production (Acemoglu & Autor, 2011). Tasks are the base units of work, that are combined as intermediate inputs to produce final output. There are three tasks: abstract (a), routine (r), and manual (m). Task specific variables are denoted by the j subindex, so that $j \in \{a, r, m\}$. The production of task j requires the labor services of occupation j and capital, which is specifically tailored to the production of that task. Technological progress is captured by A_t, which is total factor productivity, and is neutral across tasks. These are combined according to a Cobb-Douglas production function:

$$y_{j,t} = A_t k_{j,t}^{\alpha} l_{j,t}^{1-\alpha} \tag{28}$$

where $\alpha > 0$ determines the capital share in income.

Final output requires three intermediate inputs: abstract, routine, and manual tasks. These are combined according to the following constant elasticity of substitution production function, similar to Goos, Manning & Salomons (2014):

$$y_t = \left(\sum_{j \in \{a,r,m\}} \omega_j^{\sigma} y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{29}$$

where $\varepsilon > 0$ is the elasticity of substitution among tasks, $\omega_j > 0$ is the production intensity of task j, and $\sigma = (1 - \alpha + \alpha\varepsilon)/\varepsilon$ is a scaling factor due to the nested productive structure. These task intensities add up to one.

Final output can be split between consumption and investment in the three capital stocks:

$$y_t = c_t + \sum_{j \in \{a,r,m\}} i_{j,t} \tag{30}$$

Investment expenditures do not translate one-to-one into new capital. Instrad, one unit of investment converts into q_j units of capital. The capital accumulation evolves according to:

$$k_{j,t+1} = (1 - \delta) k_{j,t} + q_{j,t} i_{j,t} \tag{31}$$

where δ is the depreciation rate, and $q_{j,t}$ is the task investment specific technological level. This notion of investment specific technical change follows Greenwood, Hercowitz & Krusell (1997). Differently to them, investment specific technical change is *task specific*, which means that the efficiency to produce capital is different across tasks. This also implies that capital is task specific; it can only be used in the production of task j. Capital stocks are convertible across tasks, but the household faces different prices when doing so ($1/q_{j,t}$ units of consumption).

Labor can be allocated to the production of the three tasks. It is also perfectly mobile among tasks, and homogeneous. The total labor force is normalized to 1, so that:

$$1 = \sum_{j \in \{a,r,m\}} l_{j,t} \tag{32}$$

D.0.2 Competitive Equilibrium

In each period, the state of the economy is characterized by total factor productivity A_t, and by the task specific capital stocks $\{k_{j,t}\}_{j\in\{a,r,m\}}$ and investment specific technological levels $\{q_{j,t}\}_{j\in\{a,r,m\}}$. Their sequence of technological levels determines the sequential competitive equilibrium, which is explained in what follows.

Household

The household owns the task specific stocks of capital, which are rented at the price $R_{j,t}$. It also supplies its labor to the task producers, and earns wages $w_{j,t}$. The problem it faces is:

$$\begin{aligned}
\max_{\{c_t,\{l_{j,t},i_{j,t}\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(c_t) \\
\text{subject to:} \\
c_t + \sum_{j\in\{a,r,m\}} i_{j,t} = \sum_{j\in\{a,r,m\}} (w_{j,t} l_{j,t} + R_{j,t} k_{j,t}) \\
k_{j,t+1} = (1-\delta) k_{j,t} + q_{j,t} i_{j,t} \qquad \text{for } j \in \{a,r,m\} \\
1 = \sum_{j\in\{a,r,m\}} l_{j,t}
\end{aligned} \tag{33}$$

taking the sequence of prices $\{\{R_{j,t}, w_{j,t}\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}$ as given. Consumption goods work as the numeraire in this model.

Intermediate Task Producers

Firms producing tasks sell their product and rent capital and labor in competitive markets. Their optimization problem is:

$$\max_{k_{j,t}, l_{j,t}} \pi_{j,t} = p_{j,t} A_t k_{j,t}^{\alpha} l_{j,t}^{1-\alpha} - R_{j,t} k_{j,t} - w_{j,t} l_{j,t} \tag{34}$$

taking prices $p_{j,t}$, $R_{j,t}$ and $w_{j,t}$ as given. This is a static problem, and due to constant returns to scale and competitive markets, profits are zero.

Final Output Producers

Firms producing final output also participate in competitive markets. Their optimization problem is:

$$\max_{\{y_{j,t}\}_{j\in\{a,r,m\}}} \pi_t = \left(\sum_{j\in\{a,r,m\}} \omega_j^{\sigma} y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} - \sum_{j\in\{a,r,m\}} p_{j,t} y_{j,t} \tag{35}$$

taking task prices $p_{j,t}$ as given. As in the problem of intermediate task producers, this is a static problem where profits are zero.

Equilibrium

A sequential competitive equilibrium in this economy is a sequence of allocations $\{c_t, y_t, \{l_{j,t}, i_{j,t}, k_{j,t+1}, y_{j,t}\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}$, a sequence of prices $\{\{R_{j,t}, w_{j,t}, p_{j,t}\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}$, a sequence of task investment specific technological levels $\{\{q_{j,t}\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}$, and a sequence of total factor productivities $\{A_t\}_{t=0}^{\infty}$ such that:

1. The sequence $\{c_t, \{l_{j,t}, i_{j,t}, k_{j,t+1}\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}$ solves the household's optimization problem, taking as given the initial stocks of capital $\{k_{j,0}\}_{j\in\{a,r,m\}}$, and the sequence of prices and investment technological levels $\{\{R_{j,t}, w_{j,t}, q_{j,t}\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}$.

2. The sequence $\{\{y_{j,t}, k_{j,t}, l_{j,t},\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}$ solves the problems of intermediate task producers, taking as given the sequence of prices and total factor productivities $\{\{R_{j,t}, w_{j,t}, p_{j,t}\}_{j\in\{a,r,m\}}, A_t\}_{t=0}^{\infty}$.

3. The sequence $\{y_t, \{y_{j,t}\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}$ solves the problems of final output firms, taking as given the sequence of prices $\{\{p_{j,t}\}_{j\in\{a,r,m\}}\}_{t=0}^{\infty}$.

4. Markets clear:

$$y_t = c_t + \sum_{j\in\{a,r,m\}} i_{j,t} \tag{36}$$

$$y_{j,t} = A_t k_{j,t}^{\alpha} l_{j,t}^{1-\alpha} \tag{37}$$

$$1 = \sum_{j \in \{a,r,m\}} l_{j,t} \tag{38}$$

5. Capital stocks follow their laws of motion

$$k_{j,t+1} = (1-\delta)k_{j,t} + q_{j,t} i_{j,t} \tag{39}$$

D.1 Model Estimation

This section describes how the model is parametrized to match certain features of the data. This is done in several steps. Broadly speaking, the employment shares are informative of the relative composition of task specific capital stocks. These are leveled up using data on real income and total factor productivity. The capital accumulation process implies a path for the units of investment, that are a mixture of the resources invested, and the levels of investment specific technology $(i_{j,t} q_{j,t})$. To separate those, I use data on investment shares for a benchmark year. Finally, the intertemporal Euler equations imply the rest of the path for the levels of task investment specific technology.

At each point in time, occupational employment shares in the data determine the model's relative capital stocks. This follows from the optimality conditions when task producers decide how much labor to hire. Labor is freely mobile and homogeneous, which equalizes wages across the production of tasks. Therefore, the first order conditions for the task producers' problem (34) imply that

$$\begin{aligned} \frac{l_{i,t}}{l_{j,t}} &= \left(\frac{p_{i,t}}{p_{j,t}}\right)^{\frac{1}{\alpha}} \frac{k_{i,t}}{k_{j,t}} \\ &= \frac{\omega_i}{\omega_j} \left(\frac{k_{j,t}}{k_{i,t}}\right)^{\frac{\alpha(1-\varepsilon)}{1-\alpha(1-\varepsilon)}} \end{aligned} \tag{40}$$

With information on the production intensities, ω_j, and the capital share α and elasticity of substitution ε, this equation leads to the relative capital stocks.

The levels of capital are used to match the levels of income in the data. Solving for the intertemporal allocations of the model (the derivations are presented in appendix

??) in terms of the capital stocks yields the following expression for final output:

$$y_t = A_t \left(\sum_{j \in \{a,r,m\}} \omega_j / k_{j,t}^{\frac{\alpha(1-\varepsilon)}{1-\alpha(1-\varepsilon)}} \right)^{-\frac{1-\alpha(1-\varepsilon)}{1-\varepsilon}} \tag{41}$$

The income and total factor productivity data, that come from the Penn World Tables, are normalized to the US levels in 2000. The next logical step is to normalize the US capital stocks in that year. This serves two purposes. First, it provides a way to estimate the intensity parameters ω_j from equation (40). Second, this choice expresses the rest of the capital stocks in relative terms to US levels in 2000. These are recovered from equation (41).

Capital accumulation provides a way to measure effective investment:

$$k_{j,t+1} = (1-\delta)k_{j,t} + q_{j,t} i_{j,t} \tag{42}$$

The problem now is to disentangle investment technological levels and investment expenses from effective investment, i.e., separating the $q_{j,t} i_{j,t}$ series into $q_{j,t}$ and $i_{j,t}$. As a first approximation, I use the Euler equations:

$$\frac{c_{t+1}}{c_t} = q_{j,t}\beta \left[R_{j,t+1} + \frac{1-\delta}{q_{j,t+1}} \right] \tag{43}$$

Assuming that the growth rate of $q_{j,t}$ is the same for all tasks for a year provides a way to approximate the task investment specific technological levels:

$$q_{j,t} R_{j,t+1} = q_{j',t} R_{j',t+1} \tag{44}$$

Finally, the levels are scaled to match the investment shares in 2000.

This solves for the task investment specific technology levels in one year. The path is completed through an iterative process that uses the household's Euler equations and the resource constraints. The Euler equations inform about relative growth rates in $q_{j,t}$, and the resource constraints level that growth.

The initial estimates of $q_{j,t}$ imply as well the consumption level. Therefore, the Euler equations and the resource constraint for the following period pose a system of 4 equations in 4 unknowns:

$$\frac{c_{t+1}}{c_t} = q_{j,t}\beta\left[R_{j,t+1} + \frac{1-\delta}{q_{j,t+1}}\right] \tag{45}$$

$$c_{t+1} = y_{t+1} - \sum_{j\in\{a,r,m\}} \frac{q_{j,t+1}i_{j,t+1}}{q_{j,t+1}} \tag{46}$$

This is possible because the series of capital stocks was already estimated: it determines the rental rates $R_{j,t+1}$, output y_{t+1}, and effective investment $i_{j,t+1}q_{j,t+1}$. This process can be iterated forwards, up to the final year with information, or backwards to the initial year. This way, the entire path of task investment specific technology levels and consumption can be inferred.

The rest of the parameters are determined from the Penn World Tables, or borrowed from other studies. The capital share in income is set to 0.4, which is the average found in the data. Similarly, the depreciation rate is averaged to 0.05. The discount factor β is set to 0.95, and the elasticity of substitution is set to 0.0625, to match the estimate of Vindas (2017) in the United States.

D.1.1 Alternative identification strategy

An alternative way to disentangle initial task investment specific technological levels is explained in this section. Its actual implementation is left to future iterations of this project.

Investment is measured in different units, depending on the context. First, the investment expenses $i_{j,t}$, like those on the resource constraint (30), are denoted in units of *consumption*. Second, effective investment is measured in units of capital, which is $q_{j,t}i_{j,t}$. The relative price of investment for capital in task j is the number of units of consumption paid, per measured units of investment:

$$\begin{aligned}\frac{P^I_{j,t}}{P^C_{j,t}} &= \frac{i_{j,t}}{q_{j,t}i_{j,t}} \\ &= \frac{1}{q_{j,t}}\end{aligned} \tag{47}$$

In the data, we don't observe these individually, but rather through an aggregate investment price. This is a weighted average over investment expenditure shares, which is the first moment to target. It is given by:

$$\begin{aligned}\frac{P_t^I}{P_t^C} &= \sum_{j\in\{a,r,m\}} \frac{i_{j,t}}{\sum\limits_{j'\in\{a,r,m\}} i_{j',t}} \frac{P_{j,t}^I}{P_{j,t}^C} \\ &= \sum_{j\in\{a,r,m\}} \frac{i_{j,t}}{\sum\limits_{j'\in\{a,r,m\}} i_{j',t}} \frac{1}{q_{j,t}}\end{aligned} \tag{48}$$

The second moment is the aggregate relative price of capital. It is informative since this is another weighed average; the composition of the current capital stock is different from the composition of investment expenses. In particular, this is equal to the replacement-value weighted average of the relative prices of capital, and is given by:

$$\frac{P_t^K}{P_t^C} = \sum_{j\in\{a,r,m\}} \frac{k_{j,t}/q_{j,t}}{\sum\limits_{j'\in\{a,r,m\}} k_{j',t}/q_{j',t}} \frac{A_{t+1}^{\frac{1}{\alpha}}}{q_{j,t}} \tag{49}$$

The third and final moment is the investment rate, which is given by:

$$s_t = \frac{\sum\limits_{j\in\{a,r,m\}} i_{j,t}}{Y_t} \tag{50}$$

Intuitively, the investment and capital prices pin down the relative levels of $q_{j,t}$ across occupations, since these are two different weighted averages. The investment rate, on the other hand, defines the actual levels that are compatible with the aggregate investment decisions.

To solve this system, it has to be expressed in terms of the quantities that are available so far. This requires substituting $i_{j,t}$ with $q_{j,t}i_{j,t}/q_{j,t}$. This results in a quadratic set of equations in the task investment specific technology levels. To solve it, I log-linearize this system around 1. Its approximation is given by:

$$\ln\left(\frac{P_t^I}{P_t^C}\right) \approx -\sum_j \frac{i_{j,t}q_{j,t}}{\sum_{j'} i_{j',t}q_{j',t}}\left[2-\frac{i_{j,t}q_{j,t}}{\sum_{j'} i_{j',t}q_{j',t}}\right]\ln q_{j,t} \tag{51}$$

$$\ln\left(\frac{P_t^K}{P_t^C}\right) \approx -\sum_j \frac{k_{j,t}}{\sum_{j'} k_{j',t}}\left[2-\frac{k_{j,t}}{\sum_{j'} k_{j',t}}\right]\ln q_{j,t} \tag{52}$$

$$s_t \approx \sum_j \frac{i_{j,t}q_{j,t}}{Y_t} - \sum_j \frac{i_{j,t}q_{j,t}}{Y_t}\ln q_{j,t} \tag{53}$$

This is a linear system on log-deviations from 1, which then allows to approximate the task investment specific technological levels in 2005.